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Cardinal Vowel System to include:

2nd CVs " [y, ,oe, OE, D, c, o, u]

An additional set formulated by adding the phonetic property of plus liprounding to correspond to its new function of set operation.

Moreover, the possibility still exists to extend the basic Cardinal Vowel System to include several sets whenever one adds one of the following functional phonetic property of : nasalization or pharyngalization.

Finally, this study shows the value of the hypothesis of having some model which could be replaced by a more satisfactory one rendered through the faith in a mathematical theory; yet, other types of vagueness and insufficiencies in phonetics sometimes may encounter us which can be met with similar ways of scientific frameworks.

NOTES

(1) See Cohen: 1966, Dalon & Monna: 1982, Drake: 1974 and Morse: 1985.

(2) See op cit, P. 31 and Dalen, Doet & Swart: 1978, p. 19.

(3) See for example Abercrombie: 1971, Gimson: 1972, Heffner: 1949, Jones: 1969 & 1975, Katamba: 1989, Vassilyev: 1970 and Ward : 1972.

(4) See Awaness: 1984 & 1989, Cutler: 1985, Robins: 1981 and Yule : 1985.

(5) See Cohen: 1966, p. 9, Drake: 1974, p. 29 and Doet & Swart: 1978, p. 15.

(6) Ibid; Drake, 1974.

(7) Ibid; Awaness: 1984, Cutler: 1986, Gleason: 1961 and Robins: 1931

(8) See; Iadefoged; 1975, Vassilyev: 1970 and Yule: 1985.

(9) See; Abercrombie: 1971, p. 151, Gimson: 1972, p. 37, Ward: 1972, p. 59 and Yule, 1985, p. 43.

(10) See; Gimson: 1972, p. 76, Jones: 1969, p.18 and Ward: 1972, p.60.

(11) The two systems with six more secondary CVs fail to locate a suitable position to each point within the diagram. Their domain is rather mixed up.

(12) See; Abercrombie: 1971, Gimson: 1972, Katamba: 1989 and Ward: 1972.

(13) The validity of the diagram is considred to be another issue, therefore, it is excluded from the discussion; see Katamba: 1989; Ladefoged: 1975 and Yule: 1985.

i – The actual presentation of the concept of Cardinal Vowel System within two different sets of eight vocalic elements in each set is rather confusing and vaguely formulated.

ii –The inconsistent relational property among the total Vowels of each set that characterises its referential relationship does not coincide properly with the formation of a scientific set.

iii –The actual formation of elements of the Cardinal Vowel System represents a 'set of sets ', where two different corresponding phonetic properties are used within each set, which confuses the functional totality of the system.

iv – The functional confusion of each system can be cleared off by splitting the use of each of the phonetic property of minus lip-rounding and plus lip-rounding within each system separately.

v – It will be rather more convenient to formulate one baic Cardinal Vowel set which can be extended to include several others by adding any other functional phonetic property to the corresponding variables (elements) of the set.

The analytical revision, based on the set theory, proved to a certain extent that the actual formation of the total number of the elements of each set is rather confused. The set theoric aspects of property, totality and consistency of its functional properties do not allow two different functienal properties within the corresponding mathematical operation of all the variables of each system. The total number of variables of a system has to denote a set which can be, by all means, more convenient to present and apply The following modified system (and its set of eight vocalic variables (elements)) justifies its formation which might be more intelligible and less confused than before.

BCVs " ϵ[i, e, ξ ,a, a, $\wedge$, 6 , a]

This is a finite set of eight vocalic variables (elements) presented by one consistent phonetic property of minus lip–rounding in correspondence with its functional set theoric operation.

Relatively, one can easily extend the formation of the above basic

It is, as well as the improperly formulated set of other eight different vocalic elements, it is another 'set of sets' (Dalen, p. 13) which has to be specified :

SCVs [i, e, ,a, ,Cs, OE, D] and,
SCVs [, , ,] three elements of some other set (system) wh-i

these can be another set but not the SCVs system.

Finally, Drake: p. 34, Cohen: p. 51 and Morse: p.25 emphasize the fact that any two or more than two sets, which share one identical property in common, can be one set, i.e., a primary Cardinal Vowels set can be as follows:

PCVs " ∈ [i, e, ,a, , , ,]

Where; this is a set of eight different elements sharing in common the phonetic property of minus lip-rounding among all its variables. They rationally present a unified set which can be considered as a system of a reliable scale. Similarly,

SCVs " ∈ [y, , oe, OE, D, c, o, u]

It is the formation of a possible additional set which can be presented by adding another phonetic property (feature) to the eight vocalic elements of the above formulated Primary (or Basic) Cardinal Vowel System.

Besides, another possibility still exists to extend the primary Cardinal Vowels set into a third set, if one adds the feature of nasalization to the eight elements of the primary Cardinal Vowel set,or even a fourth set, by adding some other phonetic feature like pharyngealization,and so on.

CONCLUDING REMARKS

The discussion proved the value of utilizing the mathematical theoric apparatus to revise the phonetic data under study. The set theory was useful to reveal some confusing presentation of the variables that govern the referential relationship of all the vocalic elements in two different sets. The analytical revision carried out here in relevanvce to some abstract aspects of the set theory like property, totality and relationship revealed the following:

X1 " 1 ∈ [1, 2, 3, 4, 5]

and X2 " 2 ∈ [62 , 72 , 82 .]

Relatively, and in an attempt to utilize the previous disciplinary mathematical procedures in phonetics or phonology, it is quite normal to realize some phonological or phonetic relations presented by terms like "a member of collection ", " ——— a class " and " ——— a family ", i.e., members of sets, where the relation among their elements or constituents is similar to that one presented by one unified set; i.e., variables sharing one common property.

Therefore, if one has to revise the data of the Cardinal Vowel System, one has to specify what kind of property (feature) is used both in the formation of any linguistic set.

Hence, the revision and the validity of the data presented by the Cardinal Vowels System (sets) have to be focussed first on the accuracy of the formation of this system by sets of eight referencial elements in each.

Thus, to specify the set theoric relation of all the elements of the primary Cardinal Vowel System (PCVS), one finds the following:

PCVS " ∈ [i, e, , a, a] ɔ, o, u]]

i.e., a mixed mathematical set, because , the above set is a " set of sets " with more than one definite feature or phonetic relation is used to formulate the above set of sets, which are intuitive liprounding among the first five elements and plus liprounding among the last three elements.

Thus, and on the basis of the set theoric principles (Dalen: p. 13), the above inaccurate set has to be split into:

PCVs ∈ [i, e, , a, a] and, another set of:

PCVs ∈ [c, o, u] These are three elements of some other set (system) which can be another set but not the PCVs " set (system).

Similarly , the revision of the secondary Cardinal Vowel System (SCVI) will reveal the following:

SCVs " ∈ [y, , ɸ, OE, Ɒ [, ,]]

ent elements (numbers) sharing one common mathematicsl which is either real or natural. property

Thus, the digit 1 is a member of set A " and it cannot be a member of set B ". Similarly, the digit 12 is a member of set B " and it will cause a clear confusion to consider it a member of set A ".

Moreover, one has to distinguish between the formation of two different types of sets. A finite set is a set which comprises a limited number of elements and never accepts any other variables, e. g.,

C " set $\in$ [2, 4, 6. 8, 10] = finite set;

where C " set consists of only five elements; and

D " set $\in$ [2, 4, 6, 8, 10, ... n] non – finite set;

i.e., set D " has no limits for its elements; therefore, it is quite easy to add one or more relative members to it. In addition to this, and in order to make a clear distinction between different sets, I refer to a fundamental use of the term 'relation'in the set theory which is mostly presented by the concept "is an element of ", "is a member of" or "it belongs to". Thus , to avoid the lapses resulting from the use of all these expressions, I will restrict my discussion to the use of the following mathematical principal formula and operation.

$2 \in$ [C "], i.e., 2 is an element of set C ", and $14 \in$[D "], i.e., 14 is an element of D " set.

Finally,I would like to direct the reader's attention to the following supposition;if there are two sets whose elements are exactly those objects which have an identical property in both, they must be one set (Drake : p. 34, Cohen: p. 51 and Merse: p. 25); whereas, a set with two or more different properties is an inaccurate set and can be analysed and split into a 'set of sets'. For example,

X " $\in$ [1, 2, 3, 4, 5, [62 , 72 , 82]] incorrect set.

Where set X " contains two different elements of real and natural numbers respectivly to be split into two different sets of X1 " and X2 " i.e.,

property which represents a scientific scale of auditory equidistant measurement has to be revised on the basis of some reliable procedures. I have thought that the set theory in mathematics might help to present some adequate analytical procedure for such a revision and reformulation.

AIM AND PROCEDURE

The primary aim of this paper is to shed light on the above problem located in reference to the data presented by the Cardinal Vowel System apart from its diagram (13). Thus, the analytical procedure will cover the following axes only:

i – Revising the scientific accuracy of the data presented within a scale of two different systems by adopting some relative operations of the set theory.

ii – Presenting a revised realization of the data which might also be more efficient, less confusing and more reliable in any further studies.

DISCUSSION

First, I think it will be more convenient to the reader to formulate some workable knowledge about sets and set theory before I proceed in the revision of the problem and its data. It is quite easy to recognize and formulate mathematical sets of some common objects, elements and digits, for example, a set of 1990 cars, a set of linguistic books on the shelves and a set of real numbers ...etc. In order to simplify the case, I will take samples of mathematical sets using digits (numbers):

The digits (1, 2, 3, 4, 5, 6, ...) form a set of real numbers which can be presented in a set theoric formula by:

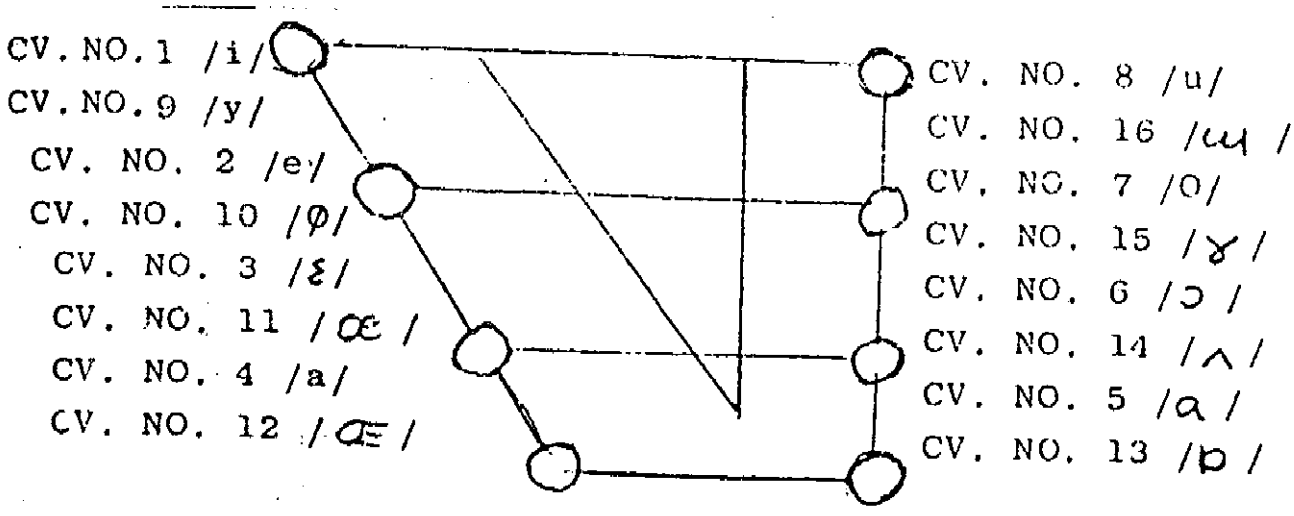
$$A \text{ " set } \in [1, 2, 3, 4, 5, 6, \ldots]$$

Where, $\in$ means contains and [...] the limits of the set variables.
Similarly, B " set represents a set of natural numbers, e.g.,

$$B \text{ " set } \in [1^2, 2^2, 3^2, 4^2, 5^2, 6^2, \ldots]$$

Where , B " set contains natural numbers and none of the real numbers. This, simply, presents the fact that we have two different mathematical sets of numbers: A " set is a set of real numbers, while B " set is a set of natural numbers. This emphasizes the fact that each set contains differ-

simply contains eight referential variables (vowels) which are believed to form a conventional scale for measurement and description. This system is called the primary Cardinal Vowel set as against the secondary Cardinal Vowel set.From the very beginning,these two sets were confused when they were presented within one identical diagram (12) of similar procedural value. It was thought that this would present a unified scale of approximate referential points of measurement for any vowel sound. The following diagram illustrates the confusion presented within Jone's Cardinal Vowels scale using numbers:



Nobody denies the importance of the above scale; it has been used for years now and its value lies in the following points:

i – The items are arbitrarily selected and generally used as descriptive devices.

ii – They are peripheral elements, their limit is always inclusive.

But, the claim that these elements constitute one system which can be used adequately by two different sets of eight related elements within each is considered the cause of a possible confusion in application and apprehension. Morcover, the vaiidity of the statement that each set is formulated by the set of eight elements of exactly determined and invariable vocalic

is very liberal (5). A set can be easily obtained by putting together a number of objects or elements sharing some common characteristic (s) (6). Dalen (1982: p. 13) defines a set as 'a collection of certain distinct objects of our intuition or of our thought into a whole'. Generally, the use of the set theory outside mathematics enforces the existence of one common feature to relate objects or elements first, then one constructs a set, which can be a finite or non-finite one.

Phonology and phonetics provide, as I think,a wealth of such related elements: vowels, consonants, features, syllables, pitches... etc. Therefore, the use of the set theory in this study is that of utility and validity. The set theory will help, as it is thought,to revise and then to reconstruct the set formation of the constituents of the Cardinal Vowel systems for I believe that the availability of the set theoric apparatus provides a healthy methodological procedure in studying some aspects which have similar phonetic features in common(7). Yet,I will not venture here to deal with the independent problems of some higher axioms of the set theory; I will stick to those parts that do not require a refined mathematical apparatus.

This study will be restricted to an analytical revision of the type of phonetic data which has to be formulated and presented in sets only. It will not question the validity of the diagram used here for the set theoric apparatus falls rather short of providing the means and the mathematical operations for such an issue (8).

THE PROBLEM

The problem is related in general to the procedure of identifying adequately many different ranges of vowel sounds,for vowel sounds vary considerably from one language to another and from one pronunciation to another (idiolects). Some phoneticians (9) were confronted with such a problem and tried to find a rationally acceptable solution for it by referring mostly to the different movements of the tongue inside the oral cavity addition to the shape and posture of the lips. Thus, the problem in particular lies in the establishment of some selected variables within sets of measurable vowel sounds to formualte what is known as the Cardinal Vowel System (10).

Various systems of Cardinal Vowels have been presented (11). But, the main concern of this study is directly related to the fundamentality of constructing these sets which represent two different scales. Each system

"REVISING THE CARDINAL VOWEL SYSTEM ON THE BASIS OF THE SET THEORY"

By

Lazim M. Al-Badawi

Dept. of Translation

College of Arts

1992

AESTRACT

This paper is an attempt to utilize the theoric operations of the set theory in mathematics.It is assumed to bring some clearness to the selection and formation of the total vocalic elements of the cardinal vowel system. Some abstract aspects of the set theory like, totality, property and functional relationship are used to help to realize some confusion and improper formation within the two sets of the system. Each of them is attested to contain vaguely a 'set of sets' which is thought to be the cause of the inconsistency and insufficiency of their application.

The revision revealed the possibility of formulating a refined basic system, which contains one intelligible set of eight different vocalic variables (elements). This basic system,unlike before, can be extended to include two or more sets, once one changes the identical relative phonetic property of all the vocalic elements of the set within the basic system.

INTRODUCTION

The set theory in mathematics is very dominant in use in many scientific fields (1). Some mathematicians even claim that the best presentation of connected scientific facts or objects has to be made in sets (2).This study is an attempt to apply the set theory in a revision to the status of the cardinal vowel system and the arrangement of its different connected elements which are actually presented within two different sets (3). Yet, one does not guarantee a total success for this attempt, but, nevertheless, it is a try to utilize mathematical devices in phonetics which may lead to a kind of refined scientific explanation of some other recurrent data. It might provide motivation for a stronger belief in the scientific supremacy of mathematics to solve some other phonotic problems (4).

It is quite obvious that the concept of a set formation in mathematics